

$$1) (x_1, x_2) + (y_1, y_2) = (x_1 + y_1, x_2 + y_2)$$

$$\textcircled{1} \quad x + y = y + x$$

$$\begin{aligned} (x_1, x_2) + (y_1, y_2) &= (x_1 + y_1, x_2 + y_2) \\ (y_1, y_2) + (x_1, x_2) &= (y_1 + x_1, y_2 + x_2) \end{aligned}$$

$$\textcircled{2} \quad x + 0 = x$$

$$(x_1, x_2) + (0, 0) = (x_1, x_2)$$

$$\textcircled{B} \quad (c_1 + c_2)x = c_1x + c_2x$$

$$\begin{aligned} ((c_1 + c_2)x_1, (c_1 + c_2)x_2) &= (c_1x_1, c_1x_2) + (c_2x_1, c_2x_2) \\ &= (c_1x_1 + c_2x_1, c_1x_2 + c_2x_2) \end{aligned}$$

$$\left[\begin{array}{ccc|c} 1 & 0 & 2 & 1 \\ 1 & 1 & 1 & 0 \\ 0 & 1 & 1 & 3 \end{array} \right]$$

$$x_1 + 2x_3 + x_4 = 0$$

$$x_1 + x_2 + x_3 + x_4 = 0$$

$$x_2 + x_3 - 3x_4 = 0$$

$$\begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} a + \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix} b + \begin{bmatrix} 2 \\ 1 \\ 1 \end{bmatrix} c = \begin{bmatrix} 1 \\ 1 \\ 3 \end{bmatrix}$$

$$\left[\begin{array}{ccc|c} 1 & 0 & 2 & 1 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & 2 & 3 \end{array} \right]$$

$$b = \frac{3}{2}$$

$$c = \frac{3}{2}$$

$$a + 2\left(\frac{3}{2}\right) = 1$$

$$a = -2$$

$$\left[\begin{array}{ccc} 1 & 0 & 2 \\ 1 & 1 & 1 \\ 0 & 1 & 1 \end{array} \right]$$

$$\left[\begin{array}{cccc} 1 & 0 & 2 & 1 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & 2 & 3 \end{array} \right]$$

$$x_4 = 1$$

$$2x_3 + 3(1) = 0$$

$$x_3 = -\frac{3}{2}$$

$$x_2 = -\frac{3}{2}$$

$$x_1 + 2\left(-\frac{3}{2}\right) + 1 = 0$$

$$x_1 = 2$$

$$N = \begin{bmatrix} 2 \\ -\frac{3}{2} \\ -\frac{3}{2} \\ 1 \end{bmatrix} a$$

$$\begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} a + \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix} b = \begin{bmatrix} 2 \\ 1 \\ 1 \end{bmatrix}$$

$\{[\circ]\}$ $\{[\vdots]\} \dots$

$$\begin{bmatrix} 2 \\ 1 \\ 1 \\ 0 \end{bmatrix} \quad \begin{bmatrix} 3 \\ 5 \\ 0 \\ 1 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 1 & 0 & a & b \\ 0 & 1 & c & d \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} 2 \\ 1 \\ 1 \\ 0 \end{bmatrix}$$

$$2 + a = 0$$

$$a = -2$$

$$1 + c = 0$$

$$c = -1$$

$$\begin{bmatrix} 1 & 0 & -2 & -3 \\ 0 & 1 & -1 & -5 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} 3 \\ 5 \\ 0 \\ 1 \end{bmatrix}$$

$$3 + b = 0$$

$$b = -3$$

$$5 + d = 0$$

$$d = -5$$